MATHEMATICAL SCIENCE

Paper - II

OCT–11/01

Signature of Invigilators

1. ....

2. ....

Roll No. ☐☐☐☐☐

(In figures as in Admit Card)

Roll No. ....

....

(in words)

Time Allowed : 75 Minutes] **[Maximum Marks : 100**

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page.
2. This paper consists of **fifty (50)** multiple choice type questions. **All** questions are compulsory.
3. Each item has upto four alternative responses marked (A), (B), (C) and (D). The answer should be a capital letter for the selected option. The answer letter should entirely be contained within the corresponding square.

   Correct method **A** Wrong method **A** OR **A**

4. Your responses to the items for this paper are to be indicated on the ICR Answer Sheet under Paper II only.
5. Read instructions given inside carefully.
6. Extra sheet is attached at the end of the booklet for rough work.
7. You should return the test booklet to the invigilator at the end of paper and should not carry any paper with you outside the examination hall.
8. There shall be no negative marking.
9. Use of calculator or any other electronic devices is prohibited.

પરીક્ષાર્થીઓ માટે સૂચનાઓ :

૧. આ પાનાની ટોચમાં દર્શાવેલી જગ્યામાં તમારો રોલનંબર લખો.

૨. આ પ્રશ્નપત્રમાં બહુવૈકલ્પિક ઉત્તરો ધરાવતા કુલ **પચાસ (૫૦)** પ્રશ્નો આપેલા છે. બધા જ પ્રશ્નો ફરજિયાત છે.

૩. પ્રત્યેક પ્રશ્ન વધુમાં વધુ ચાર બહુવૈકલ્પિક ઉત્તરો ધરાવે છે. જે (A), (B), (C) અને (D) વડે દર્શાવવામાં આવ્યા છે. પ્રશ્નનો ઉત્તર કેપીટલ સંજ્ઞા વડે આપવાનો રહેશે. ઉત્તરની સંજ્ઞા આપેલ ખાનામાં બરાબર સમાઈ જાય તે રીતે લખવાની રહેશે.

   ખરી રીત : **A** ખોટી રીત : **A** અથવા **A**

૪. આ પ્રશ્નપત્રના જવાબ આપેલ ICR Answer Sheet ના Paper II વિભાગની નીચે આપેલ ખાનાઓમાં આપવાના રહેશે.

૫. અંદર આપેલ સૂચનાઓ કાળજીપૂર્વક વાંચો.

૬. આ બુકલેટની પાછળ આપેલું પાનું રફ કામ માટે છે.

૭. પરીક્ષા સમય પૂરો થઈ ગયા પછી આ બુકલેટ જે તે નિરીક્ષકને સોંપી દેવી. કોઈપણ કાગળ પરીક્ષા ખંડની બહાર લઈ જવો નહીં.

૮. ખોટા જવાબ માટે નેગેટિવ ગુણાંકન પ્રથા નથી.

૯. કેલ્કયુલેટર અને ઈલેક્ટ્રોનિક યંત્રોનો પ્રયોગ કરવાની મનાઈ છે.

MATHEMATICAL SCIENCE

PAPER-II

Note : This paper contains **FIFTY (50)** multiple-choice/Assertion and Reasoning Matching questions. Each question carrying **two (2)** marks. Attempt **All** the questions.

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1. The infinite series $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{n^{\alpha}}$ is :

   (A) convergent if $\alpha > \frac{1}{2}$

   (B) convergent only if $\alpha > 1$

   (C) convergent only if $\alpha > \frac{3}{2}$

   (D) divergent if $0 < \alpha < 1$

2. Let $x_n = 1 + \frac{(-1)^n}{n}$ for $n \in \mathbf{N}$.

   Then,

   (A) $\limsup_{n\to\infty} x_n = \frac{3}{2}$

   $\liminf_{n\to\infty} x_n = 0$

   (B) $\limsup_{n\to\infty} x_n = \liminf_{n\to\infty} x_n = 1$

   (C) $\limsup_{n\to\infty} x_n = 1$

   $\liminf_{n\to\infty} x_n = 0$

   (D) $\limsup_{n\to\infty} x_n = 1$

   $\liminf_{n\to\infty} x_n = -1$

3. Which of the following subsets of $\mathbf{R}$ is countable and dense in $\mathbf{R}$?

(A) $\mathbf{Z}$

(B) $\{a + b\sqrt{2} \mid a, b \in \mathbf{Q}\}$

(C) The set of all irrational numbers

(D) The set of all real numbers with 0 in the fourth decimal place

4. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be defined by

$$f(x, y) = \begin{cases} 0 & \text{if} \quad xy = 0 \\ 1 & \text{if} \quad xy \neq 0 \end{cases}$$

Then :

(A) $f_x(0, 0), f_y(0, 0)$ exist and f is continuous at (0, 0)

(B) $f_x(0, 0), f_y(0, 0)$ exist but f is not continuous at (0, 0)

(C) f is differentiable at (0, 0)

(D) The partial derivatives of f do not exist at (0, 0)

5. Let $f : \mathbf{R} \to \mathbf{R}$ be defined by,

$f(x) = x^3 + x + a$, where $a \in \mathbf{R}$

Then :

(A) $f(x)$ has no real root for any a

(B) $f(x)$ has exactly one real root for all a

(C) $f(x)$ has one real root if $a \geq 0$ and three real roots if $a < 0$

(D) $f(x)$ has three real roots for all values of a

6. $f : [0, 100] \to \mathbf{R}$ is defined by,

$f(x) = [x]$, (where $[x]$ denotes the greatest integer less than or equal to x)

Then :

(A) The upper integral $\int_{0}^{-100} f(x)\,dx = 5050$

(B) The lower integral $\int_{-0}^{100} f(x)\,dx = 4900$

(C) The upper integral $\int_{0}^{-100} f(x)\,dx$

= The lower integral $\int_{-0}^{100} f(x)\,dx = 4950$

(D) The lower integral $\int_{-0}^{100} f(x)\,dx = 5000$

7. Which of the following subsets of $\mathbf{R}^2$ is *not* connected under usual metric ?

(A) $\{(x, y) \in \mathbf{R}^2 : xy = 0\}$

(B) $\{(x, y) \in \mathbf{R}^2 : 1 < x^2 + y^2 < 3\}$

(C) $\{(x, y) \in \mathbf{R}^2 : xy = 1\}$

(D) $\{(x, y) \in \mathbf{R}^2 : x^2 + y^2 = 1\}$

8. Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $P = \begin{pmatrix} 3 & 3 \\ 8 & 9 \end{pmatrix}$. Let $B = P^{-1} AP$. Then :

(A) tr A = tr B and det A = det B

(B) tr A = tr B but det A ≠ det B

(C) tr A ≠ tr B but det A = det B

(D) tr A ≠ tr B and det A ≠ det B

9. Let V be the set of real functions $y = f(x)$ satisfying $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$. Then dim V =

(A) 4 (B) 3

(C) 7 (D) 2

10. In the vector space C[0, 1] :

(A) $\{x^3, \sin x\}$ is linearly independent but $\{x^3, \sin x, \cos x\}$ is linearly dependent

(B) $\{x^3, \cos x\}$ is linearly independent but $\{\cos x, \sin x\}$ is not linearly independent

(C) $\{x^3, \sin x, \cos x\}$ is linearly independent but $\{x^2, \sin x, \cos x\}$ is linearly dependent

(D) $\{x^3, \sin x\}$ is linearly independent

11. The system of equations

$$\begin{aligned} x - 3y + z &= -1 \\ 2x + y - z &= 2 \\ 4x - 5y + z &= 0 \end{aligned}$$

has :

(A) exactly one solution

(B) at least 3 but a finite number of solutions

(C) an infinite number of solutions

(D) no solution

12. If T is an operator on $\mathbf{R}^2$ carrying a line l to l, then :

(A) $\dim T(\mathbf{R}^2) = 1$

(B) $\dim T(\mathbf{R}^2) = 2$

(C) T carries line parallel to l to line perpendicular to l

(D) T carries line parallel to l to a line parallel to l

13. The unitary matrix P such that $P\begin{bmatrix} 2 & i \\ -i & 2 \end{bmatrix}P^* = \begin{bmatrix} 3 & \\ & 1 \end{bmatrix}$ is :

(A) $\begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix}$

(B) $\begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix}$

(C) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix}$

(D) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix}$

14. Let V be a finite dimensional vector space, and S be a subset of V that spans V, L be a linearly independent subset of L. Then :

(A) $|S| \geq \dim V$; $|L| \geq \dim V$

(B) $|S| \leq \dim V$; $|L| \leq \dim V$

(C) $|S| \leq \dim V$; $|L| \geq \dim V$

(D) $|S| \geq \dim V$; $|L| \leq \dim V$

15. The function $\log(\cos \pi z)$ has :

(A) a simple pole at $\frac{1}{2}$

(B) a removable singularity at $\frac{1}{2}$

(C) an essential singularity at $\frac{1}{2}$

(D) a branch point at $\frac{1}{2}$

16. The function $\sin z$ maps :

(A) horizontal lines other than x-axis to parabolas

(B) horizontal lines other than x-axis to ellipses

(C) vertical lines other than y-axis to ellipses

(D) vertical lines other than y-axis to parabolas

17. Let $f(z) = z^3 - 3z = u(x,y) + i\,v(x, y)$. Then the value of $\det\begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix}$ at $z = i + \sqrt{2}$ equals :

(A) 75 (B) 63

(C) 64 (D) 72

18. If the function $f(z) = \dfrac{(1-\cos z)^k}{z^7}$ has a simple pole at the origin, then the value of k is :

(A) 1 (B) 2

(C) 3 (D) 4

19. The residue of $\dfrac{\sin z}{z^4 - 1}$ at $z = i$ equals :

(A) $\frac{1}{4}\cos 1$ (B) $\frac{1}{4}\cosh 1$

(C) $\frac{1}{4}\sinh 1$ (D) $-\frac{1}{4}\sinh 1$

20. Which of the following natural numbers cannot be written as a sum of two squares ?

(A) 101 (B) 141

(C) 265 (D) 113

21. Let G be a simple group of order 168. Then the number of elements of order 7 in G is :

(A) 12 (B) 48

(C) 24 (D) 30

22. The maximum possible order of an element of the symmetric group S_8 is :

(A) 15 (B) 12

(C) 8 (D) 24

23. For a positive integer n, let $U(n) = \{\bar{x} \mid 1 \le x \le n, (x, n) = 1\}$ denote the group of prime residue classes modulo n under multiplication. Then :

(A) U(8) and U(10) are isomorphic groups

(B) U(10) and U(12) are isomorphic groups

(C) U(8) and U(12) are isomorphic groups

(D) No two groups of U(8), U(10) and U(12) are isomorphic

24. Which of the following fields are isomorphic ?

(A) $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{3})$ (B) $\mathbb{Q}(i)$ and $\mathbb{Q}(\sqrt{-2})$

(C) $\mathbb{Q}(2^{1/3})$ and $\mathbb{Q}(\pi)$ (D) $\dfrac{\mathbb{Q}[x]}{(x^2+1)}$ and $\mathbb{Q}(i)$

25. The last two digits of 19^{42} are :

(A) 61 (B) 41

(C) 81 (D) 21

26. $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of

$$(1-x^2)\,y'' - 2xy' + p(p+1)\,y = 0; \; -1 < x < 1.$$

Given that $y_1(0) = 1$, $y_1'(0) = 0$, $y_2'(0) = 1$, the value of the Wronskian $W(y_1, y_2)$ at $x = \frac{1}{2}$ is :

(A) $\frac{4}{3}$ (B) $\frac{1}{2}$

(C) $\frac{3}{4}$ (D) 2

27. The characteristic curves in (x, t) plane for $x u_x + t u_t = 0$ are :

(A) rays starting from the origin

(B) circles in the (x, t) plane with centre at the origin

(C) family of lines parallel to x-axis

(D) family of lines parallel to t-axis

28. Which of the following is *true* ?

(A) The wave equation and the Laplace's equation are hyperbolic

(B) The wave equation and the Laplace's equation are elliptic

(C) The Laplace's equation is elliptic and the wave equation is hyperbolic

(D) The Laplace's equation is hyperbolic but the wave equation is elliptic

29. For a forced harmonic oscillator $\ddot{x} + \omega^2 x = F(t)$, $(\omega \neq 0)$, with periodic forcing function $F(t)$, it is known that all solutions are periodic with same period as $F(t)$. Then which of the following is a possible candidate for $F(t)$?

(A) $F(t) = \cos \omega t$ (B) $F(t) = 1$

(C) $F(t) = \cos 2\omega t$ (D) $F(t) = \sin \omega t$

30. Given a (3×3) matrix A with real entries such that whenever $u(x)$ is a harmonic function of (x_1, x_2, x_3), the function $v(x) = u(A\,x)$ is also harmonic. Then, A can be :

(A) Any (3×3) non-singular matrix

(B) Any (3×3) symmetric matrix

(C) Any (3×3) orthogonal matrix

(D) Any (3×3) matrix with determinant 1

31. The Fredholm integral equation

$$\phi(x) - \lambda \int_0^1 \cos(x-t)\,\phi(t)\,dt = 0$$

has :

(A) No eigen values

(B) At least one real eigen value

(C) Infinitely many eigen values with a finite limit point

(D) An eigen value λ_0 corresponding to which there are infinitely many eigen vectors

32. Let $\vec{r}$ be a position vector of a point moving under the action of a force $\vec{F}$.

If $\vec{F}$ is parallel to the negative z-axis and $\vec{L} = \frac{d\vec{r}}{dt} \times \vec{r}$, then :

(A) the z-component of $\vec{L}$ is constant

(B) $|\vec{L}|$ is constant

(C) all the components of $\vec{L}$ are constants

(D) none of the components of $\vec{L}$ remain constant

33. Given distinct real numbers $x_1, \ldots\ldots\ldots, x_n$; let

$$L_j(x) = \frac{(x - x_1)\ldots\ldots\ldots(x - x_{j-1}).\,(x - x_{j+1})\ldots\ldots\ldots(x - x_n)}{(x_j - x_1)\ldots\ldots\ldots(x_j - x_{j-1})\,(x_j - x_{j+1})\ldots\ldots\ldots(x_j - x_n)}$$

$j = 1, 2, \ldots\ldots, n$, be the Lagrange polynomials. Let $Q(x) = x^3 - 2x + 4$. Then

$\sum_{j=1}^{n} Q(x_j)\, L_j(3)$ equals :

(A) 23 (B) 25

(C) 27 (D) 29

34. The initial value problem

$$\frac{dy}{dx} = |x| + \sqrt{\left|\cos\left(\frac{\pi x}{2}\right)\right|}\ ; \ y(a) = 0$$

has :

(A) a unique solution in a neighbourhood of a if $a = 0$

(B) infinitely many solutions in a neighbourhood of a if $a = 0$

(C) a unique solution in a neighbourhood of a if $a = 1$

(D) finitely many solutions in a heighbourhood of a if $a = 1$

35. The integral of function $f(x) = 2|x| + |x - 2|$ over the interval $[-1, 1]$ is computed using trapezoidal rule with partion points of step size (0.2). The difference between the computed value and actual value is :

(A) 0.2 (B) –0.4

(C) 0 (D) –0.2

36. The ODE $y'' + y = x \cos x$ is solved by the method of undetermined coefficients. The form of the particular integral is :

(A) $A \cos x + B \sin x$

(B) $A x \cos x + B x \sin x$

(C) $A x^2 \cos x + B x^2 \sin x$

(D) $A x^2 \cos x + B x^2 \sin x + C x \cos x + D x \sin x$

37. If a is an integer and if

$$d = \frac{1}{3}a^3 + \frac{1}{5}a^5 + \frac{1}{7}a^7 + \frac{34}{105}a,$$

then :

(A) $d \in \mathbf{N}$

(B) $d \in \mathbf{Z}$

(C) $d \in \mathbf{R} - \mathbf{Q}$

(D) $d \in \mathbf{Q} - \mathbf{Z}$

38. The least squares regression line is the line :

(A) which is determined by use of a function of the distance between the observed Y's and the predicted Y's

(B) which has the smallest sum of the squared residuals of any line through the data values

(C) for which the sum of the residuals about the line is zero

(D) which has all of the above properties

39. In single-facrtor ANOVA, MSTr is the mean square for treatments, and MSE is the mean square for error. Which of the following statements are *not* true ?

(A) MSE is a measure of between samples variation

(B) MSE is a measure of within samples variation

(C) MSTr is a measure of between samples variation

(D) The value of MSTr is affected by the status of (true or false)

40. For any two events A and B, which of the following are always *correct* ?

(A) P (A or B) = P (A) + P (B)

(B) P (A or B) = P (A) . P (B)

(C) P (A or B) = P (A) + P (B) – P (A and B)

(D) P (A and B) = P (A) . P (B)

41. Which of the following statements are true for the following data values :

9, 7, 8, 6, 9, 10 and 14 ?

(A) The mean and median are euqal

(B) The mean is larger than the median

(C) The mean is smaller than the median

(D) The mean and the 10% trimmed mean are equal

42. A travel agent has a list of 400 regular customers. Of these, 80 have been to Delhi and 100 to Mumbai. There are 20 who have been to both Delhi and Mumbai. The probability that are person selected at random from the list has not been either to Delhi or to Mumbai is :

(A) 0.45 (B) 0.50

(C) 0.60 (D) 0.55

43. Let x and y be independent N (0, 1) random variables. What will be the distribution of $\frac{(x+y)^2}{(x-y)^2}$?

(A) Cauchy

(B) t-distribution

(C) F-distribution

(D) Normal distribution

44. If X has discrete uniform distribution on 0, 1, 2, ..., m and the mean of the distribution is 10. Then the value of m is :

(A) 10 (B) 15

(C) 20 (D) 25

45. Let a sample space S be the open interval (0, 1). Let events A, B and C be given by the interval (0.3, 0.6), (0.5, 0.8) and (0, 0.2) respectively. If the probability of an event is equal to the length of the interval, which of the following statements is FALSE ?

(A) A and B are not independent events

(B) $A \cup B$ and $B \cup C$ are exhaustive events

(C) B and C are mutually exclusive but not independent

(D) $P(A' \cup B') = 0.9$

46. Let Y_1, Y_2, Y_3 and Y_4 be independent standard exponential variables. The distribution of $\frac{Y_1 + Y_3}{Y_2 + Y_4}$ is same as that of aX where :

(A) $a = 6$ and $X \sim \beta_2 (2, 2)$

(B) $a = 1$ and $X \sim \beta_2 (2, 2)$

(C) $a = 6$ and $X \sim F (2, 2)$

(D) $a = 6$ and $X \sim F (4, 4)$

47. Let X be a random variable with cdf

$$F(x) = \begin{cases} 0 & , \quad x \le 0 \\ 1 - e^{-x}, & x > 0 \end{cases}$$

The value of $P\left(\frac{1}{4} \le e^{-x} \le \frac{1}{3}\right)$ is :

(A) $\frac{1}{12}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $e^{-3} - e^{-4}$

48. Let X and Y be two independent discrete random variables with E(X) = μ and E(Y) = λ. Let the distribution of X given X + Y = 5 is binomial with parameters 5 and ½. Then :

(A) X and Y have binomial distribution with parameters (2, ½) and (3, ½)

(B) X has binomial distribution *b* (2, ½) and Y has Poisson distribution with λ = 3

(C) X and Y have Poisson distribution with λ = μ

(D) X and Y have Poisson distribution with λ = 2μ

49. Consider the following statements in respect of the two types of errors (α, β) of a test procedure :

(*i*) It is not possible to minimize α and β simultaneously, since one generally increase with the decrease of the other

(*ii*) In the classical tests, greater protection is given to α

Which of the above statements is/are *correct* ?

(A) (*i*) only

(B) (*ii*) only

(C) Both (*i*) and (*ii*)

(D) Neither (*i*) nor (*ii*)

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50. Match the following lists :

List 1		List 2
I.	$P\left(\bigcup_{j=1}^{n} A_j\right) \le \sum_{j=1}^{n} P(A_j)$	(*a*) Minkowski inequality
II.	$E(xy)^2 \le E(x^2)\,E(y^2)$	(*b*) Holder's inequality
III.	$\sum_{i=1}^{n} x_i y_i \le \left(\sum_{i=1}^{n} x_i^{\,p}\right)^{1/p} \left(\sum_{i=1}^{n} y_i^{\,p}\right)^{1/p}$	(*c*) Chebeshev's inequality
IV.	$\left[\sum (x_i + y_i)^p\right]^{1/p} \le \left(\sum_{i=1}^{n} x_i^{\,p}\right)^{1/p} + \left(\sum_{i=1}^{n} y_i^{\,p}\right)^{1/p}$	(*d*) Doob's inequality

The *correct* match is :

(A) I-(*c*), II-(*a*), III-(*d*), IV-(*b*)

(B) I-(*d*), II-(*a*), III-(*b*), IV-(*c*)

(C) I-(*b*), II-(*c*), III-(*d*), IV-(*a*)

(D) I-(*d*), II-(*c*), III-(*b*), IV-(*a*)

ROUGH WORK

ROUGH WORK

ROUGH WORK

SEAL